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demonstrate the difference
between providing
pre-processed mathematics
for students and
rich contexts for exploring
mathematical ideas.

athematics in textbooks and indeed in conventional classrooms is often presented as exercises or worksheets in which the mathematics itself has been processed into a form that is easily digested. This McDonald's version of mathematics ensures that the mathematical skill or technique is laid bare and typically the sole focus of attention. In this paper the mathematical focus is directed number, though the reader will soon become aware that the children's activity spans a rich panoply of disciplines. By directed number, we refer to positive and negative numbers. In this context, McDonald's mathematics might take the form of an exercise in which the children are presented with a series of additions of one negative number to another.

Ainley and Pratt (2002) have argued that, as a result of this approach, mathematics learning often becomes sterile. Children gain no sense of the bigger picture and activity is not driven by the task itself. Many primary school teachers recognise the need to promote tasks in which children construct a purpose for the activity. However, Ainley and Pratt point out that in itself the construction of purpose is insufficient. The challenge for teachers is to find tasks that, in contrast to the McDonald's approach, lead to a sense of utility for the mathematics.

When children work on tasks that encompass meaningful contexts, the mathematical ideas have necessarily to sit along-side a range of knowledge and ideas drawn from everyday experience and knowledge from other disciplines. Such knowledge is double-edged since it may either support the learning or act as a distraction. Students may over-generalise either on the basis of everyday knowledge brought into the mathematical problem, or because they apply rules blindly,

having failed to make those rules truly meaningful.

The following advice, which seems to go some way towards advocating the McDonald's approach to mathematics teachers, is taken from the Department for Education and Skills TeacherNet website (2004):

The structured nature of mathematical knowledge suits a structured teaching style. Break down content into relatively small chunks and ensure that students have fully mastered each one before going on to the next step. This will build students confidence about their ability... Children easily develop misconceptions about the meaning of mathematical concepts. Primary school pupils will often acquire a rule and then overgeneralise it to situations in which it is not applicable.

How can students possibly learn what is salient when they are confronted with a diet of McDonald's mathematics? Perhaps only by embracing context can children begin to discriminate between the salient and the irrelevant. Nevertheless, we must embrace context in an artful way if the purpose of the task is to lead to mathematical utility.

We worked for about an hour with each of two separate groups of 6 eight year-old children. The researcher, labelled M below, was an experienced teacher, who used herself as a resource for the children. In particular the researcher, in her teaching role, structured the activity, enabling collaborative and disciplined activity.



Figure 1. The screen.



Figure 2. The map.

Father Christmas's epic journey

The children were told that they were going to plan Father Christmas's journey on Christmas Eve as he delivered presents to various countries. We will describe the task in terms of the four phases identified during the analysis. In the first phase of the task, the children could use a Father Christmas character to trigger information on a computer about different countries1. The computer screen showed a country, its average temperature on Christmas Eve and a picture of Father Christmas wearing clothes appropriate to that temperature

1. The Father Christmas character contains a small RFID tag, which activates information about the country when it comes in contact with sensor strips on the map. This study is part of a European project, called Webkit: Intuitive physical interfaces to the WWW (IST-2001-34171), which is exploring the use of Tangible User Interfaces (TUIs) in education.

(Figure 1). The children wrote that temperature onto a matching card. These temperatures ranged from Nigeria, 27° C to Svalbaard, -30° C. Later the children were asked to order the cards from hottest to coldest in order to create Father Christmas's journey. In the second phase of the task, the children were asked to locate the countries on a map that was placed over sensor strips in such a way that each strip fell below the corresponding country (Figure 2). In the third phase, the children were asked to create an alternative journey for Christmas by reference to the map. In the final phase, the children were asked to write appropriate temperatures onto the cards for the countries chosen for the alternative journey. It is worth noting that the children were unable to use the computer to trigger this information since the technology was designed only to work for the preplanned countries. Thus, the children were required to create directed numbers (by which we mean positive and negative numbers) rather than use those generated on the computer.

Introducing the resources

As the children matched the strips to the cards and ordered them, they used an interesting mix of formal and informal knowledge. For example, we see how Cliona (Cl) was using the information on the cards about Father Christmas's clothes to make sense of the temperature information:

[Fiona places Father Christmas on a strip and the display shows Nigeria]

Cl: It's not very hot for just wearing pants.

M: 27 degrees is really hot

S: It is.

Children also brought to this activity significant personal knowledge and experience about countries they had visited. For example, David (D) and Liam (L) shared their holiday experiences when discussing Greece:

D: It's hot because it's near Cyprus, isn't it?

M: Well done, it's very close to Cyprus.

L: And near to Turkey

Also, the children already had some knowledge about temperatures, which they used to help them to assimilate the new information. Here four children are discussing the temperature in Poland:

D: Zero wow!

F: Zero?

S: Zero! How cold is that?

Cl: Freezing!

In this first phase, the children were simply making sense of the directed numbers representing the temperatures of the countries. In the next phase they began to use this information.

Locating the journey

The map was now available to the children: it had been laid over the sensor strips and the small model of Father Christmas triggered a screen display through the map when placed on one of the countries along the initial route from Nigeria to Svalbaard. The children used their knowledge of numbers to locate the countries in as much that their knowledge of numbers helped them know whereabouts on the map they should look for a country in relation to the other countries along the route. They did not, however, only use their knowledge of numbers: knowledge about countries, maps and temperatures as well as other personal knowledge was also drawn upon. At one point, Cliona is trying to locate Hungary:

M: Is it going to be this side of Father Christmas or the other side of Father Christmas?

Cl: There.

M: Yeah. Why do you think you should be looking in this part?

Cl: Because it's going up

M: And that means the temperature's?

Cl: Colder

Support from the teacher helps Cliona to locate the country. Cliona also needed the map, her knowledge of the temperature of Hungary and Greece and knowledge about numbers. The map itself was used almost constantly throughout this phase of the activity, sometimes by only one or two children at a time, often by most (even all) of the group. The map was used to support communication:

M: Do you know where Greece is?

S: Yes.

M: Can you see where you're going?

F: They just keep going. [She points at the map 3 or 4 times in progressively northward direction.]

Fiona's gesticulation at the map is used to show the sense of the journey. Generally this type of reference to the map helped the children to share information, test ideas, and formulate questions and answers.

One child, Marie, had somewhat idiosyncratically taken ownership of the cards, laying them out in front of her on the table in order. She interacted with the map much less than the other children, but was quick to supply information about the countries and temperatures and became a resource for the group. We mention this not only to emphasise the complex interrelationship between knowledge of numbers and the map, screen and cards, but also to highlight how

different children create their own personal approach to using resources. Marie's use of the cards was quite different from others in the group but she nevertheless performed an important role in the collaboration. Furthermore the use of knowledge was not limited to numbers; in locating countries, children called upon personal knowledge of those countries and recently acquired knowledge about latitude.

Creating a journey

In this phase, children created a new journey using their new knowledge about maps. They did not concern themselves with the numbers but with the relationship between latitude and temperature. The challenge set by the researcher-asteacher was 'to work out a return route from the North Pole to somewhere hot enough that he just needs his swimming trunks on'.

These children were able to use prior knowledge about features of maps and globes in this phase. David demonstrated his understanding of a link between latitude and temperature when Lily (L) chose Saudi Arabia as one stop in Father Christmas's journey:

- L: Saudi Arabia.
- D: That's hot. Is it?
- M: Well, do you think it might be warmer or colder?
- D:
- M: Why do you think it's going to be warmer?
- D:
- M: I think you're right but why do you think so?
- S: Because it's farther down from the North Pole
- D: Yeah.

In this excerpt David seemed to be starting to understand the connection between south and warmer but he still needed the help of his most able friend Simon (S). Later David showed more mastery of this idea when he tried to choose a country after (and hotter than) Somalia:

- D: Then you could go to Kenya.
- F: And I'm going to Kenya.
- D: Can we finish in Kenya?
- M: Is Kenya hotter than Somalia?
- D: It goes to the side though.
- M: Yes?
- D: You could go to Tasmania. [D actually means Tanzania.]

David used the map to see which countries satisfy the conditions he was looking for. He knew that he must find a country that was further south than Somalia. Kenya bordered Somalia south and west and, though David thought that it was generally south and therefore likely to be hotter than Somalia, he was not completely confident in this choice because parts of Kenya 'go to the side' of Somalia. He was more comfortable with a country that was wholly south of Somalia and so suggested Tanzania. In fact he used the rule-of-thumb: 'farther from the North Pole means hotter'.

Estimating temperatures

In this fourth phase, the children were challenged to propose temperatures for the countries in their new journey. This was a mathematically rich phase, marked by the children's use of their knowledge of the map to interpolate and extrapolate directed numbers.

There were many examples of interpolation. In trying to estimate the temperature of Russia, Carl, Lily and Simon were able to interpolate with negative numbers.

- M: Can you remember what Finland and Poland are?
- I know Finland... Um minus L:
- C: Minus 12 yeah, so...
- M: Finland is minus 12 and Poland is zero so what do you think Russia might be?
- S: Ooh, 5.
- About 5? M:
- Ma:
- M: Yeah, I think 5 or 6 - something in between zero and minus 12
- I think it's 5 or 6. L:
- Is it 6 or minus 6? M:
- S: Minus 5 or 6.
- C: Minus.

In this example, the children were using the map, alongside their knowledge of the temperatures of adjacent countries, to estimate the temperature of an intermediate country. However, the children were able to call upon a range of knowledge. Sense-making activity is punctuated by the triggering of unexpected and sometimes unhelpful pieces of knowledge, what we might call naïve understandings.

An interesting example began when the teacher asked them to estimate the temperature for Spain. Spain is a very popular place for families to take their summer holidays, so several of the children think of Spain as very hot.

- A: It is, it is... very hot.
- L: About 15?
- L: 15, yeah 15.
- E: 15 or 16.
- M: Right, so do we know anything about any of the countries?
- E: Portugal's hot, Spain's hot and France is hot and they are all next door neighbours so Spain's...

They used the map but only to confirm what they knew about Spain, forgetting that this epic journey was to take place in winter. Only through the teacher's efforts to steer the children towards using the latitudes on the map as the relevant knowledge rather than their summer holidays, were the children able to recognise an apparent contradiction. Here is another example of the use of naïve understanding taking place:

- J: 'Cause that's bigger so it's going to have more area in it.
- E: So probably... colder.
- M: You think the bigger countries do what?
- J: Are going to be colder because more air is going to spread around 'cause like when you've had water in something.

- A: Yeah, but look at England.
- J: It goes really cold after it's hot.
- A: Yeah but James, England is colder than Spain.
- E: Oh yeah... and that's really small isn't it?
- A: Yeah
- E: And that's a lot colder than Chad.
- A: Yeah.
- M: So does that mean, James, that your sort of logical thinking there has got something wrong with it?
- J: I think I'm wrong.

We do not know the source of James's (J) rule-of-thumb that 'larger countries are colder', but the activity with the map and feedback from his peers, allowed James to question this piece of knowledge.

We have seen in the final phase, how the children forged connections between the map and their knowledge of temperatures to interpolate and extrapolate new knowledge, triggering along the way other pieces of knowledge, which needed to be accounted for in the sense-making activity.

What did we learn?

Analysis of the children's activity in these four phases showed changes in patterns, starting from a position in which the children worked with numbers and connected that knowledge to maps in particular. The example of Marie (with the cards), however, reminds us yet again that only the inconsistency of children is consistent!

Later the role of the map was important. Even then this is an oversimplification; other types of knowledge were either reinforced or occasionally denied during the activity. In particular, we recognised the use of geographically-oriented pieces of knowledge, some of which were obscure in origin.

Without doubt the drive for this activity came from a perception of the task as engaging and purposeful, in the sense of Ainley and Pratt (2002). The children could relate to the task at an age where Father Christmas still holds some magic. However, this engagement is insufficient. The task was intended as one that would bring together the geographical concept of latitude and the mathematical concept or skill of interpolating directed number. We observed the construction of two utilities, the first for directed number and the second for latitude:

- directed numbers *inter alia* were used to locate countries;
- knowledge of the relative position of countries on the map inter alia was used to interpolate and extrapolate directed numbers.

Father Christmas's epic journey was successfully designed in order that the purpose of the task led almost inevitably to the construction of these two utilities. We believe that by paying attention to purpose and utility in designing sense-making tasks, teachers can avoid the McDonald's approach and provide their children with a rich and stimulating diet.

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Note

A more researched focused version of this paper, 'Numbers and maps: The dynamic interaction of internal meanings and external resources in use', was published in the conference proceedings of MERGA 27, *Mathematics Education for the Third Millennium, Towards 2010*, Townsville, June 2004.

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